Achievements of Ms. Disha A. Shah (lecturer, Mathematics, CTIDS – Wadhwancity, AY – 2014-15

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CERTIFICATE

This is to certify that Prof./~~Dr.~~/~~Mr.~~/Ms. ***Disha A. Shah*** ~~Participated~~ / presented a poster in the ***"XXIX Gujarat Science Congress 2015"*** organized by Gujarat Science Academy at the Science City, Ahmedabad, during February 28 and March 1, 2015.

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~~Dr.~~/~~Mr.~~/Ms. Disha A. Shah *of* C.T.I.D.S., Wadhwan *has ~~attended~~/presented Research paper titled* Solution of one Dimensional wave Equation using Laplace Transform *at One Day National Conference RTMA -2014 on 19 December, 2014 organized by the Department of Mathematics, C.U.Shah University, Wadhwan City (Gujarat).*

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Solution of One Dimensional Wave Equation using Laplace Transform

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Abstract: *In this paper the equation of motion for the string under certain assumption has been derived which is in the form of second order partial differential equation. The governing partial differential equation represents transverse vibrating of an elastic string which is known as one dimensional wave equation. The analytical solution has been obtained using Laplace Transform.*

Keywords: Partial Differential Equation, Wave equation, Laplace Transform, Transverse

1. Introduction

In this paper, we deal with initial boundary value problem that is a second order Partial Differential Equation that occurs frequently in many physical phenomena which is known as one dimensional wave equation. The governing equation represents transverse vibrating of an elastic string. An analytical solution obtained by using Laplace Transform.

The solution of wave equation was one of the major mathematical problems of the mid eighteenth century. The wave equation was first derived and studied by D'Alembert in 1746. It also attracted the attention of Euler (1748), Bernoulli (1753) and Lagrange (1759). Solution was obtained in several different forms in series of papers. The major points at issue concerned the nature of a function and the kind of functions that can be represented by trigonometry.

2. Statement of the Problem

Consider a uniform elastic string of length l stretched tightly between two fixed points O and A, and displayed slightly from its equilibrium position OA. The line OA joining the points origin O and $A(l, O)$ is taken as the x – axis and a perpendicular line through O as the y – axis. The problem is to determine the vibrations of the string, that is, to find its deflection $y(x, t)$ at any point x and at any time $t > 0$ (see figure 1)

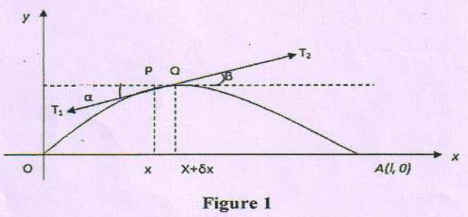

Figure 1

3. Physical Assumptions

We assume the following.

1) The vibrations are lateral and take place in a plane
2) The string is perfectly elastic and does not offer any resistance to bending.
3) The tension in the string is so large that the action of the gravitational force on the string can be neglected.
4) The displacement y and the slope $\frac{\partial y}{\partial x}$ are small, so that their higher powers may be neglected.

4. Mathematical Formulation

Let m be the mass per unit length of the string. Consider the motion of an infinitesimal elements PQ of length δs. The mass of this element is $m\delta s$, its acceleration is $\ddot{y}$ and the forces acting on it are the tensions T_1 and T_2 as shown in Figure 1.

Since there is no motion in the horizontal direction, we have

$$T_1 cos\alpha = T_2 cos\beta = T = constant \quad (1)$$

By Newton's second law of making the equation of motion in the vertical direction is

$$m\delta s \frac{\partial^2 y}{\partial t^2} = T_2 sin\beta - T_1 sin\alpha \quad (2)$$

$$\frac{m\delta s}{T}\frac{\partial^2 y}{\partial t^2} = \frac{T_2 sin\beta}{T_2 cos\beta} - \frac{T_1 sin\alpha}{T_1 cos\alpha} \quad (3)$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{T}{m\delta s}(tan\beta - tan\alpha) \quad (4)$$

Since $\delta s = \delta x$ to a first approximation and

$$tan\alpha = \left(\frac{\partial y}{\partial x}\right)_x, tan\beta = \left(\frac{\partial y}{\partial x}\right)_{x+\delta x} \quad (5)$$

$$\Rightarrow \frac{\partial^2 y}{\partial t^2} = \frac{T}{m}\left[\frac{\left(\frac{\partial y}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial y}{\partial x}\right)_x}{\delta x}\right] \quad (6)$$

As $Q \to P, \delta x \to 0$

$$\Rightarrow \frac{\partial^2 y}{\partial t^2} = \frac{T}{m}\lim_{\delta x \to 0}\left[\frac{\left(\frac{\partial y}{\partial x}\right)_{x+\delta x} - \left(\frac{\partial y}{\partial x}\right)_x}{\delta x}\right] \quad (7)$$

$$\Rightarrow \frac{\partial^2 y}{\partial t^2} = \frac{T}{m}\frac{\partial}{\partial x}\left(\frac{\partial y}{\partial x}\right) \quad (8)$$

$$\Rightarrow \frac{\partial^2 y}{\partial t^2} = \frac{T}{m}\left(\frac{\partial^2 y}{\partial x^2}\right) \quad (9)$$